Fiscal Policies in the Metropolitan Area

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July 2009

Abstract

In this paper, we apply the tax competition model to the metropolitan area which is composed of two cities: a central city and a surrounding city. All the residents are employed by the firms that are located in the central city and obtain their income from them. The purpose this paper is as follows. First, we compare the local governments' tax policies both in the closed economy and open economy. Second, we investigate how the agglomeration effects influence the migration in equilibrium, and examine the centripetal forces and the centrifugal forces in the metropolitan area. Finally, we investigate whether the competitive equilibrium in the metropolitan area produces the same result as the Pareto optimality.

The main results are as follows; each government has a dominant strategy in determining tax rate in a closed economy. And the population distribution depends on the housing lot's and public good's elasticity of utility in an open economy. The tax rates of both cities depend on the marginal productivity of the public good in the central city. Finally, the less concentration to the central city is occurred in equilibrium; furthermore, the efficiency of the public good's provision depends on the productivity of public good in the central city, too.

***JEL Classification*: H77, R13, R51**
Key Words: decentralization, tax competition, migration, metropolitan area

1. Introduction

It is necessary to reexamine the role of central government and local government along with the steady progress of decentralization. Although numerous studies have examined the decentralization problem, most of them have paid attention only to the following two points: the problem of provision of public goods, and financial problems faced by local governments. The role of local government has grown further in the provision of public goods because the range of benefits of local public goods is limited to the region. When local government supplies public goods in the region, the residents' preferences toward public goods in the region can be reflected more accurately.

Simultaneously, the tax authority of local government has been strengthened, too. Decentralization enables the provision of public goods that suit local features by transferring authority and fiscal resources to local governments. It is also typically promoted along with deregulation of finance to local governments, that is, the independent fiscal resources principle becomes necessary and indispensable.

Tax competition can be understood in such a tendency. In the field of public finance, a lot of researchers and policy makers have paid attention to the mobility of factors like capital or labor. The traditional tax competition literature since Wilson (1986) and Zodrow and Mieszlowski (1986) have argued that the mobility of tax sources as capital leads to an inefficiently low capital tax and under provision of local public good.

This kind of tax competition model has been extended variously in local public finance. Applying to the spatial model of metropolitan areas was discussed by Braid (2000). He considered the spatial model of business tax competition with two jurisdictions in the metropolitan area introducing positive commuting cost. And he suggested the following; if a lump sum tax can be used, then other taxes are not used, and the provision of local public good becomes efficient. In the case that source based wage tax and capital tax can both be used, only the former is used, and the local public good is underprovided. Guo (2009) showed the tax competition model in which local governments can use a wage tax or a land tax to finance public expenditure and compete for commuters. He insisted that the efficiency of the public good may be achieved, depending on the production function and the combination of population and land area. Glazer et al. (2008) investigated the effects of income distribution between rich and poor under the free mobility and land heterogeneity, and argued that tax competition can lead both jurisdictions to tax the rich more heavily when rich can migrate.

On the other hand, Roos (2004) and Fernandez (2005) introduced the agglomeration economies into the tax competition model. Roos (2004) asserted that a strategic government will choose a higher tax rate than a non-strategic government. He also asserted that the strategic tax rates are not Pareto optimal and local public good becomes overprovision. Fernandez (2005) analyzed tax competition model with agglomeration effects, and compared it to the traditional model, without agglomeration effects. Fernandez (2005) insisted that both models with and without agglomeration effects bring underprovision of the public good and the underprovision is more severe when agglomeration effects are present.

In this paper, we apply the tax competition model to the metropolitan area which is composed of two cities: a central city and a surrounding city. All the residents in the metropolitan area have their job opportunity at the CBD of central city. We consider that the public good which is provided by central city benefits not only the residents but also to the firms. Moreover, it benefits the residents of the surrounding city since it affects the workers' income. This is similar to Guo (2009) who considered the public good as a factor of production of the composition good, for not only the residents but also the commuters can benefit from the public good through the higher wages brought by it.

The purpose this paper is as follows. First, we compare the local governments' tax policies both in the closed economy and open economy. Second, we investigate how the agglomeration effects influence the migration in equilibrium, and examine the centripetal forces and the centrifugal forces in the metropolitan area. Finally, we investigate whether the competitive equilibrium in the metropolitan area can be attained the Pareto optimality.

This paper is organized as follows. In the next section, we introduce the basic model of this paper. Section 3 describes the market equilibrium under the introduced basic model. Moreover, we investigate the government's tax policy in the closed economy and the feature of migration in the open economy using specific functional form in section 4. Section 5 derives the Nash competition on income tax and investigates the efficiency of the provision of local public good through numerical example. Finally, section 6 presents some conclusions and further research.

2. The Model

We consider a metropolitan area which is composed of two cities: a central city and a surrounding city, which are labeled respectively as 1 and 2. The firms, residents and the government in each city compose the metropolitan economy. We assume that the firms are only located in the city 1 and the residents in the metropolitan area are considered to be labor suppliers. All the residents are employed by the firms that are located in the city 1 and obtain their income from them. Moreover, the land in each city is assumed to be owned by an absentee landlord. Each city government levies a proportional income tax from each resident. It provides public good using those fiscal resources. Here, the spillover effect of public goods between cities is assumed not to exist.

A representative firm produces homogeneous consumption good x with labor n only as input. The consumption good is a numeraire good and the price is normalized to unity. In the city 1, the public good G_1 benefits not only the residents in the city but also the firms, in that it has an impact on the productivity. The production function of the representative firm j to be given as

$$x_j = f(G_1)n_j .$$

$f(G_1)$ refers the external economy that benefits all the firms in the city 1 so that the marginal production of public goods is positive, $f'>0$. We also assume that $f(G_1)$ is concave, $f''<0$.

All residents have identical utility functions, defined over consumption good x_i, land for housing h_i and public goods G_i (i=1, 2). Thus, the utility function is given by

$$u^i = u(x_i, h_i, G_i), \qquad i = 1,2$$

Each resident in the metropolitan area earns income w by provision of one unit of labor to the firm inelastically. Since all the firms are located in the city 1, there are no job opportunities in the city 2. As the residents in the city 2 commute to the city 1, time cost are necessary for the residents to commute between both cities. Their income is reduced by the opportunity cost of time spent commuting, so their real income becomes τw[1]. De Bartolome et al. (2004) and Brock et al. (2009) introduced this kind of commuting cost which is defined as $(1-\tau)w$, $(\tau > 0)$. The intra-city commuting cost is ignored. Each city government levies proportional income tax from the residents in the city for the provision of public good. We assume the residence-based tax and the tax rate is t_i. Therefore, the budget constraints of the residents in both cities are written as,

$$(1-t_1)w = x_1 + R_1 h_1,$$
$$(\tau - t_2)w = x_2 + R_2 h_2$$

where R_i is the land rent[2].

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[1] $\tau(0 < \tau < 1)$, which can be viewed as representing the convenience of access to the central city if we denote the cost as $(1-\tau)$. That is, a traffic infrastructure between cities is maintained well when τ approaches 1.

[2] Subscript 1 refers the variables related to the central city; subscript 2 indicates those for the surrounding city.

3. Market Equilibrium

We consider a two-stage game situation. In the first stage, both governments determine the tax rate and the supply level of public good. The firms and residents determine their economic behavior under the tax rates and public goods.

3.1 Firms

A representative firm which locates in the city 1 produces the consumption good x with labor in the metropolitan. Since we assume the full employment in the labor market, the choice variable of the firm, n, becomes total population in the region, N. Moreover, the profit of a representative firm becomes zero because that the market is competitive. Therefore, the wage, w, becomes

$$w = f(G_1). \tag{1}$$

3.2 Residents

The residents in both cities behave so that their utility level may become the maximum under the budget constraint. The necessary condition for the utility maximization is,

$$\frac{\partial u / \partial h_i}{\partial u / \partial x_i} = R_i, \qquad i = 1, 2.$$

Using these conditions and budget constraints, we can obtain the following,

$$x_1 = x_1\left((1-t_1)w, R_1, G_1\right), \tag{2}$$

$$h_1 = h_1\left((1-t_1)w, R_1, G_1\right), \tag{3}$$

$$x_2 = x_2\left((\tau - t_2)w, R_2, G_2\right), \tag{4}$$

$$h_2 = h_2\left((\tau - t_2)w, R_2, G_2\right). \tag{5}$$

3.3 land market

Each city has a perfectly inelastic supply of land, which is determined exogenously by the absentee landlord. Land rent in each city is, thus, determined competitively by the demand. We fix the land supply in each city to be L_1 and L_2. As we assume that there is no open space in the each city, $L_i = n_i h_i$, (i=1,2) should be satisfied in the land market. With these conditions and Eqs. (3), (5), the land rents of each city are obtained as,

$$R_1 = R_1\left((1-t_1)w, G_1, n_1, L_1\right), \tag{6}$$

$$R_2 = R_2\left((\tau-t_2)w, G_2, n_2, L_2\right). \tag{7}$$

3.4 The wage and public goods

Since fiscal resources of public good are procured from the income tax, the size of public goods is, $G_i = t_i w n_i$. From this constraint and Eq. (1), the provision of public goods in the city i is

$$G_i = f(G_1)t_i n_i, \qquad i = 1, 2.$$

Therefore the provision level of public good in the city 1 is determined as,

$$G_1 = G_1(t_1, n_1). \tag{8}$$

Moreover, we can obtain the provision level of public good in the city 2 as follows,

$$G_2 = G_2(t_2, n_2, G_1(t_1, n_1)). \tag{9}$$

From the conditions above, the endogenous variables $\{x_i, h_i, R_i, w, G_i\}$ are determined in the market.

From Eqs. (1) and (8), we can confirm the following relations,

$$\frac{\partial G_1}{\partial t_1} = \frac{n_1 w}{D} > 0$$
$$\frac{\partial G_1}{\partial n_1} = \frac{t_1 w}{D} > 0$$
$$where \;\; D = 1 - t_1 n_1 \frac{\partial f}{\partial G_1} > 0$$

As we assume that $f(G_1)$ is concave, D is obviously positive. It means that the marginal cost of public good is larger than the marginal tax revenue from the marginal increase of public good. Moreover, from Eq. (9), we obtain

$$\frac{\partial G_2}{\partial t_1} = t_2 n_2 \frac{\partial f}{\partial G_1}\frac{\partial G_1}{\partial t_1} > 0,$$
$$\frac{\partial G_2}{\partial n_2} = t_2 w - t_2 n_2 \frac{\partial f}{\partial G_1}\frac{\partial G_1}{\partial n_1}$$

It is obvious that the effect of t_2 on G_2 is positive. And the effect of t_1 on G_2 is also positive because $\partial G_1/\partial t_1 > 0$. The increasing t_1 leads to the improvement of productivity of firms through the increasing of G_1, as a result the wage is raised. Thus G_2 is also increased. The effect of n_2 on G_2 is ambiguous since it causes the trade off relation between marginal tax revenue from population increase and marginal decrease of the wage from the decrease of population in the city 1.

4. Behavior of each government: the case of Cobb-Douglas function

In this section, we investigate the tax policy of each government. We consider the case of closed economy that the residents are immobile and extend to the open economy that the residents are mobile. The indirect utility function is useful to analyze here, which can be derived from Eqs. (1)-(9).

In the remainder of this paper, a specific functional form is chosen for production function and utility function. At first, the production function is defined as

$$x_j = n_j G_1^b, \quad 0 < b < 1.$$

Also the utility function is defined as,

$$u_i = x_i^{\alpha} h_i^{\beta} G_i^{\gamma}, \quad \alpha + \beta = 1, \quad 0 < \gamma < 1.$$

From the utility maximization problem, the size of consumption goods and land for the residents are given as,

$$x_1 = \alpha(1 - t_1)w, \quad h_1 = \frac{\beta}{R_1}(1 - t_1)w$$
$$x_2 = \alpha(\tau - t_2)w, \quad h_2 = \frac{\beta}{R_2}(\tau - t_2)w \tag{10}$$

The land rent can be obtained through the land constraint, $L_i = n_i h_i$, (i=1,2) and Eq. (10).

$$R_1 = \beta \frac{(1-t_1) w n_1}{L_1}, \quad R_2 = \beta \frac{(\tau - t_2) w n_2}{L_2} \tag{11}$$

In the firms' behavior, the size of labor demand, n, correspond to the total population, N, because full employment is assumed. In addition, from the zero profit condition, the wage, w, can be given as,

$$w = G_1^{\,b} \tag{12}$$

With Eq. (12) and budget constraint of public goods, the followings are obtained.

$$w = t_1^{\frac{b}{1-b}} n_1^{\frac{b}{1-b}}, \tag{13}$$

$$G_1 = t_1^{\frac{1}{1-b}} n_1^{\frac{1}{1-b}}, \tag{14}$$

$$G_2 = t_1^{\frac{b}{1-b}} n_1^{\frac{b}{1-b}} t_2 n_2. \tag{15}$$

The effect of t_1 on G_1, G_2, w is positive as we have seen in the section 3. Substituting Eqs. (13), (14) and (15) into Eqs. (10), (11), indirect utility function of each city can be given as,

$$V_1 = \left(\alpha(1-t_1) w\right)^{\alpha} \left(\frac{L_1}{n_1}\right)^{\beta} G_1^{\gamma}, \quad V_2 = \left(\alpha(\tau - t_2) w\right)^{\alpha} \left(\frac{L_2}{n_2}\right)^{\beta} G_2^{\gamma} \tag{16}$$

4.1 Closed economy

In a closed economy, the population of each city, n_1, n_2 is exogenously given because no migration is allowed to occur. Each government only has to decide a tax rate so that the welfare of the city may become the maximum. Since residents in the city are identical, the welfare function of each city can be defined as the utility level of the representative resident, respectively. It means that each government decides the tax rate so that the utility level of it's resident is maximized, $\partial V_i / \partial t_i = 0$. Differentiating Eq. (16) with respect to t_1, t_2, respectively, we can obtain[3]

$$\frac{\partial V_1}{\partial t_1} = V_1 \frac{1}{(1-b)t_1}\left[\gamma - \frac{\alpha(b-t_1)}{1-t_1}\right] = 0,$$

$$\frac{\partial V_2}{\partial t_2} = V_2\left[\frac{\gamma}{t_2} - \frac{\alpha}{\tau - t_2}\right] = 0$$

Two solutions exist in this problem; corner solution (t_1=1, t_2=τ) and inner solution. In the case of corner solution, the disposable income of the residents becomes zero. Excluding corner solutions, the tax rate of each city becomes

$$t_1 = \frac{\alpha b + \gamma}{\alpha + \gamma} \quad for \quad 0 < t_1 < 1, \tag{17}$$

$$t_2 = \frac{\tau\gamma}{\alpha + \gamma} \quad for \quad 0 < t_2 < \tau \tag{18}$$

Eqs. (17) and (18) shows that tax rate in the city 1 is higher than that of city 2. When determining the tax rates, both governments do not consider the counterpart's policy; i.e. in the closed economy, the tax rate of each city is determined as the welfare is maximized regardless of the tax rate of the other city[4].

The utility levels of both cities need not necessarily be corresponding in the closed economy because that the residents of both cities are immobile. Using Eqs. (17), (18), the following relation can be derived.

$$V \equiv \frac{V_1}{V_2} = \left(\frac{1-b}{\tau}\right)^{\alpha}\left(\frac{n_2 L_1}{n_1 L_2}\right)^{\beta}\left(\frac{(\alpha b + \gamma)n_1}{\tau\gamma n_2}\right)^{\gamma} \tag{19}$$

If the utility level of city 1 is higher (lower) than that of city 2, it becomes V> (<) 1.

[3] The second order condition is also satisfied.

[4] The expenditure share of housing lot, β ,does not concerns here, since n_i and L_i are exogenous variables, thus housing lot h_i becomes constant regardless of t_i in equilibrium in the closed economy.

Various factors affect on V. From Eq. (19), we can obtain the following,

$$\frac{\partial V}{\partial b} < 0, \quad \frac{\partial V}{\partial \tau} < 0,$$
$$\frac{\partial V}{\partial n_1} \geq (<) 0 \ \ if \ \ \beta \leq (>) \gamma$$

Under Eqs. (17) and (18), if b or τ increases, V_2 becomes high, and V_1 becomes high along with the increase of n_1 if $\beta>\gamma$. These relations do not mean, however, that $V>1$ or $V<1$. For example, τ is sufficiently high, V_2 is higher than V_1 regardless of b ($V|_{b=0} < 1$). Fig. 1 shows this[5].

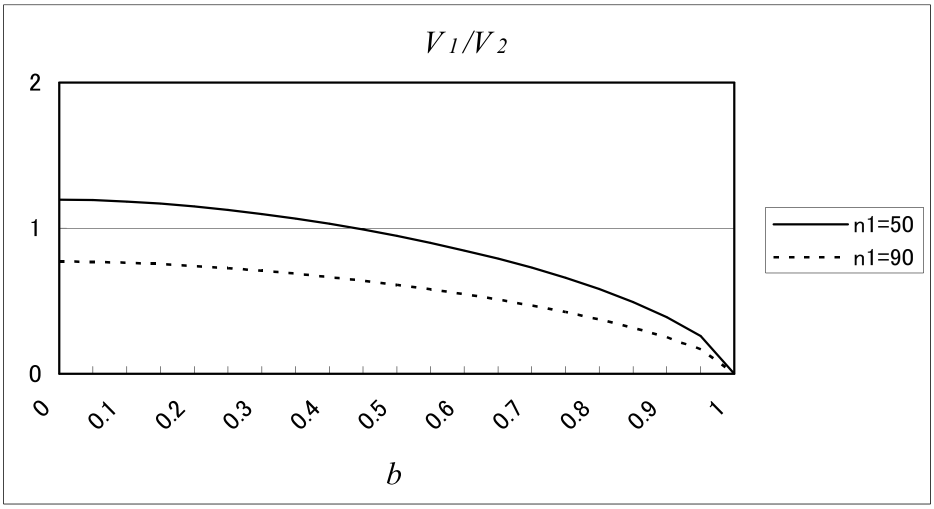

Fig. 1 comparison of utility levels in the closed economy

When determining the provision level of G_1, the government of city 1 must consider both firms and residents. If the productivity by the public good is high, the government should raise the provision level. As a result, the tax burden of residents in city 1 is increased even though the income w is also increased. Eq. (17) shows this. On the other hand, the tax burden of the residents in city 2 does not depend on b. Therefore, if b is sufficiently high, V_2 becomes higher than V_1 even though the population is symmetric.

We summarize the results of this subsection in proposition.

Proposition 1. *In the metropolitan area in which the economy is closed,*

1. *The tax rate of city 1 is higher than that of city 2.*

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[5] α=0.6, β=0.4, γ=0.3, τ=0.8, L_1=100, L_2=100, N=100.

2. *Each city's welfare maximization tax rate is determined independently regardless of the other city's tax rate. It means that each government has dominant strategy on determining its tax rate.*
3. *If β or τ becomes high, V_2 becomes high. If β is higher (lower) than γ, V_2 becomes high (low) along with the increase of n_1.*

4.2 Open economy

In the open economy, the migration equilibrium is attained when $V_1=V_2$. It is necessary to check the stability of equilibrium because the shapes of V_1, V_2 with n_1 is not clear. Differentiating V_1/V_2 with respect to n_1, we can obtain

$$\frac{d(V_1/V_2)}{dn_1}=\frac{N}{n_1 n_2}\left[\left(\frac{n_2 L_1}{n_1 L_2}\right)^{\beta}\left(\frac{t_1 n_1}{t_2 n_2}\right)^{\gamma}\left(\frac{1-t_1}{\tau-t_2}\right)^{\alpha}(\gamma-\beta)\right] \tag{20}$$

The condition for the stability is that Eq. (20) should be negative. Therefore, the following inequality should be satisfied.

$$\beta>\gamma \tag{21}$$

If γ is larger than β, it is obtained the corner solution in which all the residents reside one city. The slope of Eq. (20) depends on β and γ. β is the parameter which indicates the preference for housing lot and γ, public good. β and γ can be interpreted as the housing lot's and public good's elasticity of utility, respectively. This result is similar to Roos (2004). He considers that the public good and the tax on the value of housing are centripetal forces while competition for the fixed housing stock is the centrifugal force. He showed that downward sloping of Eq. (20) is achieved when the centrifugal force is stronger than the centripetal one. Differ from Roos (2004), we consider that γ as the centripetal force, yet β is considered as centrifugal force, which is the same as Roos (2004).

Fig. 2 shows the stability of migration equilibrium. If γ becomes higher, the line rotates to upper right and the inner solution becomes unstable.

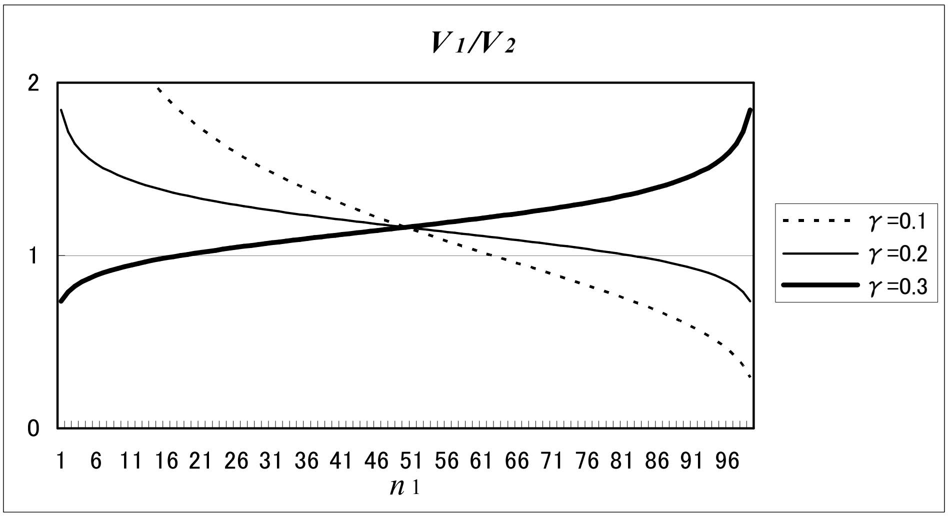

Fig. 2 The stability of migration equilibrium[6]

The effect of tax rate t_i on the migration is not determined uniquely. The population of each city in equilibrium can be obtained when V_1/V_2=1. From the indirect utility of both cities, n_1 is obtained as,

$$n_1 = \frac{N}{1+Q},$$
$$where\ \ Q = \left[\left(\frac{1-t_1}{\tau - t_2}\right)^{\alpha}\left(\frac{L_1}{L_2}\right)^{\beta}\left(\frac{t_1}{t_2}\right)^{\gamma}\right]^{\frac{1}{\gamma-\beta}} \tag{22}$$

Differentiating Eq. (22) with respect to t_1 and t_2, respectively, we can obtain the followings.

$$\frac{\partial n_1}{\partial t_1} = \frac{QN((\alpha+\gamma)t_1 - \gamma)}{t_1(1-t_1)(\gamma-\beta)(1+Q)^2}$$
$$\frac{\partial n_1}{\partial t_2} = \frac{QN(\tau\gamma - (\alpha+\gamma)t_2)}{t_2(\tau-t_2)(\gamma-\beta)(1+Q)^2}$$

Assume that $0<t_1<1$ and $0<t_2<\tau$, the effect of tax rates on the population of city 1 depends on α and γ.

[6] All the parameters are the same values as Fig. 1, except $t_1 = t_2 = 0.1$.

$$t_1 \geq (<) \frac{\gamma}{\alpha+\gamma} \Rightarrow \frac{\partial n_1}{\partial t_1} \leq (>) 0 \tag{23}$$

$$t_2 \geq (<) \frac{\tau\gamma}{\alpha+\gamma} \Rightarrow \frac{\partial n_1}{\partial t_2} \geq (<) 0 \tag{24}$$

When t_1 is lower than the threshold ($\gamma/(\alpha+\gamma)$), the rise of t_1 has a positive effect on the increasing n_1. If the tax burden in the city 1 is too low, the increase of wage and public good through the increase of tax burden makes city 1 attractive, thus the population in city 1 increase. However, the tax burden exceeds a certain level, the effects are reversed and some residents in city 1 migrate to city 2. The same effects can be said as t_2.

We summarize the results of this subsection in proposition.

Proposition 2 *In the metropolitan area in which the economy is open,*

2.1 The population distribution depends on the housing lot's and public good's elasticity of utility, (β *and* γ)*. If* $\beta>\gamma$*, the unique inner solution exists at* $0<n_1<N$ *in equilibrium. However, if* $\beta<\gamma$*, the inner solution in equilibrium is unstable and the corner solutions exist* ($n_1=0$ *or* $n_1=N$).

2.2 When the tax rate of each city is low, the rise of t_i *has a positive effect on the increasing the population in the city. However, it exceeds a threshold, a negative effect is occurred. The threshold in city 2 coincides with the tax rate in closed economy, and that of city 1 is lower than the tax rate in closed economy.*

In the open economy, each government decides the tax rate, t_i so that its welfare may become maximum constraint to Eq. (22). It is different from a closed economy because that each government should consider the tax rate of counterpart's government in open economy. Thus, each government has incentive to behave strategically. We will investigate this through numerical analysis in the next section.

5. Numerical analysis

5.1 Tax competition

In this subsection, we investigate each government's tax policy in open economy. Each government decides the provision level of public good and tax rate respectively that maximizes the utility of its residents (Eq. (16)) constraint to utility equalization condition (Eq. (22)). Differ from the case of closed economy, each government reacts the counterpart's policy in open economy. The first order condition of utility maximization problem of both governments are given as,

$$\frac{\partial V_1}{\partial t_1} = \left.\frac{\partial V_1}{\partial t_1}\right|_{n_1} + \frac{\partial V_1}{\partial n_1}\frac{\partial n_1}{\partial t_1} = 0 \tag{25}$$

$$\frac{\partial V_2}{\partial t_2} = \left.\frac{\partial V_2}{\partial t_2}\right|_{n_1} + \frac{\partial V_2}{\partial n_2}\frac{\partial n_2}{\partial t_2} = 0 \tag{26}$$

Eqs. (25) and (26) represent the governments reaction functions in implicit form. Though the signs of each term are not obvious, we can confirm the following considering $\beta>\gamma$ from Eq. (21),

$$\frac{\partial V_1}{\partial n_1} = V_1\left(\frac{b-(\beta-\gamma)}{(1-b)n_1}\right)$$

$$\frac{\partial V_2}{\partial n_2} = V_2\left(\frac{-b}{1-b}\frac{\alpha}{n_1} + \frac{\gamma-\beta}{n_2}\right) < 0$$

The effect of n_1 on V_1 depends on the size of $b,\ \beta,\ \gamma$.

$$b \geq (<)(\beta-\gamma) \Rightarrow \frac{\partial V_1}{\partial n_1} \geq (<)0. \tag{27}$$

If $b=(\beta-\gamma)$, the second term of Eq. (25) becomes zero. It means that the government of city 1 decides t_1 without depending on t_2, because $\partial V_1/\partial t_1|_{n1}$ dose not depends on t_2. Thus the tax rate of city 1 in open economy is corresponding to that in closed economy (Eq. (17)). As t_1 is given to city 2 in this case, the government of city 2 should decide t_2, which satisfies Eq. (26). However, t_2 in equilibrium does not determined uniquely. We confirm multiple solutions between $t_2 \in (0,\ 0.4)$ through numerical analysis.

On the other hand, if $b \neq (\beta-\gamma)$,there exist a unique Nash equilibrium. From Eqs. (13), (14) and (15), we can confirm the effects of b on G_1, G_2, w as follows,

$$\frac{\partial w}{\partial b} = \frac{w}{(b-1)^2}\log(n_1 t_1) > 0 \qquad if \quad n_1 t_1 > 1,$$
$$\frac{\partial G_i}{\partial b} = \frac{G_i}{(b-1)^2}\log(n_1 t_1) > 0 \qquad if \quad n_1 t_1 > 1, \qquad i = 1,2$$

If the productivity by the public good, b becomes high, firm's marginal benefit of public good rises, too. This brings the increase of public good and wage. The utility levels of both cities are also improved by the increase of G_1, G_2 and w. As we seen at Fig. 1, however, V_1/V_2 falls along with b. The rising effect of b on V_2 is lager than that on V_1, that is, b is centrifugal force in this paper. Thus the population of city 1, n_1, decreases with b.

Therefore, t_1 in open economy is lower (higher) than that in a closed economy when $b > (<)(\beta - \gamma)$. From Eq. (27), if the effect of n_1 on V_1 is positive, the government has an incentive to lower the tax rate. This is corresponding to the traditional tax competition. However, if the effect is negative, city 1 determines the tax rate at a relatively high rate while city 2 determines it a lower rate regardless of the change b. If b rises, t_1 in closed economy becomes high. In the open economy, however, it restricts the government's options since the residents of city 1 emigrate, t_1 is lower under the level of closed economy. We give a numerical example to show this. If $\beta = 0.4,\ \gamma = 0.2,\ b = 0.1$, the tax rates of both cities in open economy are t_1=0.339, t_2=0.1999 where tax rates in closed economy are t_1=0.325, t_2=0.2. If $\beta = 0.4,\ \gamma = 0.2,\ b = 0.3$, the tax rates of both cities in open economy are t_1=0.441, t_2=0.1999 where tax rates in closed economy are t_1=0.475, t_2=0.2.

We summarize the results of this subsection in proposition.

Proposition 3 *In the metropolitan area in which the economy is open,*

3.1 The government of city 1 has dominant strategy in tax competition, but the tax rate of city 2 is ambiguous when $b = (\beta - \gamma)$.

3.2 While the tax rate of city 2 is lower than that in closed economy regardless of b, the tax rate of city 1 is lower (higher) than that in closed economy when $b > (<)(\beta - \gamma)$.

Proposition 3 is due to the economic structure of the metropolitan area that we considered in this paper. If the surrounding city has firms and some residents are hired there, we should take into consideration the difference of productivity between two cites. Roos (2004) insisted that a strategic government will choose a higher tax rate than a nonstrategic government when both cities have firms. If we consider, however, that the surrounding city has an agriculture industry and its production is constant, then we can set it as zero and this situation corresponds to our model.

5.2 Optimal allocation

In this subsection, we compare the competitive equilibrium allocation and the socially optimal allocation. To investigate the optimal allocation, we apply the optimal land use model in Fujita (1989) that maximizes surplus subject to a set of prespecified target utility level, instead of the Benthamite welfare function. As we ignored the intracity commuting cost, the total cost in the metropolitan area can be defined as the sum of composite consumption good costs, public good costs and opportunity land costs. We assume the opportunity land cost as zero so that the optimal allocation problem can be defined as,

$$\begin{aligned}
\max_{\{x_i, h_i, G_i, n_i\}} \quad & (n_1 + \tau n_2) f(G_1) - n_1 x_1 - n_2 x_2 - G_1 - G_2 \\
s.t. \quad & u(x_1, h_1, G_1) = U \quad :(\lambda_1) \\
& u(x_2, h_2, G_2) = U \quad :(\lambda_2) \\
& L_1 = n_1 h_1 \quad :(\lambda_3) \\
& L_2 = n_2 h_2 \quad :(\lambda_4) \\
& N = n_1 + n_2 \quad :(\lambda_5)
\end{aligned}$$

where λ_i indicates the Lagrange multiplier. U refers the target utility level, and we assume U as the utility level in equilibrium. From the first order conditions, we can obtain

$$\begin{aligned}
& (n_1 + \tau n_2) \frac{\partial f}{\partial G_1} + n_1 \frac{\partial u / \partial G_1}{\partial u / \partial x_1} = 1, \\
& n_2 \frac{\partial u / \partial G_2}{\partial u / \partial x_2} = 1,
\end{aligned} \tag{28}$$

$$f(G_1) - x_1 - h_1 \frac{\partial u / \partial h_1}{\partial u / \partial x_1} = \tau f(G_1) - x_2 - h_2 \frac{\partial u / \partial h_2}{\partial u / \partial x_2} \tag{29}$$

Eq. (28) indicates Samuelson condition that the marginal benefits of the public good equal to marginal cost. Eq. (29) refers the optimal population distribution condition. On the other hand, the shadow rent of the land, λ_3 and λ_4 are

$$\lambda_3 = \frac{\partial u / \partial h_1}{\partial u / \partial x_1}, \qquad \lambda_4 = \frac{\partial u / \partial h_2}{\partial u / \partial x_2}. \tag{30}$$

Eq. (30) is corresponding to Eqs. (6) and (7). It means that the shadow rent under the target utility coincide with the equilibrium rent, that is, if the utility level and the population distribution between market equilibrium and optimal allocation are the same, the land rental market becomes efficient.

Table 1 shows the results of numerical analysis.

	$b<(\beta\text{-}\gamma)$		$b>(\beta\text{-}\gamma)$	
	Equilibrium	Optimal allocation	Equilibrium	Optimal allocaton
n_1	69.399681	85.290564	64.08753	96.21046
w	1.420139	1.4156242	4.186412	4.862578
$V_1(=V_2)$	1.6546532	1.6546532	3.805503	3.805503
R_1	0.2607634	0.3716169	0.600181	1.000263
R_2	0.1042961	0.0474316	0.360827	0.024657
x_1	0.5636123	0.6535604	1.404754	1.559492
x_2	0.51125	0.4836855	1.507108	0.975983
h_1	1.4409288	1.1724626	1.560366	1.039388
h_2	3.2679398	6.7983573	2.784548	26.38839
t_1	0.338548	0.2676892	0.440749	0.416357
t_2	0.199999	0.1138922	0.199999	0.066904
G_1	33.366339	32.320626	118.2515	194.7849
G_2	8.6913413	2.3715802	30.06888	1.232843
t_1-*closed*	0.325		0.475	
t_2-*closed*	0.2		0.2	

Table 1 Numerical analysis: equilibrium and optimal allocation

Like the case of market equilibrium, we should consider two cases in which $b<(\beta-\gamma)$ and $b>(\beta-\gamma)$. In both cases, the population of city 1 in equilibrium (n_1^e) is less than that in optimal allocation (n_1^{opt}). It means that the central city is less concentrated in the market equilibrium. This brings the distortions of consumption of goods and the land rent market in the metropolitan area. The less populated city 1 leads to the residential cost ($R_i h_i$, i=1,2) which is less than its optimal cost in city 1, while it is larger than its optimal cost in city 2. If b is changed from $b<(\beta-\gamma)$ to $b>(\beta-\gamma)$, a higher concentration occurs in city 1, so n_1 is increased in the optimal allocation. However, the effect of b on n_1 is negative in the market equilibrium (see Fig. 1). Therefore, the increasing b brings more severe distortion of population distribution in the metropolitan area.

In the case of $b<(\beta-\gamma)$, both governments take the higher tax rate than the optimal level, and results in the overprovision of public goods. On the other hand, in the case of $b > (\beta - \gamma)$, the wage is lower than that in optimal allocation, and the population distribution distortion becomes more significant. Thus, an underprovision of the public good in city 1 occurs even though the tax rate is higher than in optimal allocation. The results of this subsection are summarized as the following proposition.

Proposition 4 *In the metropolitan area in which the economy is open,*

4.1 The less concentration to the central city is occurred in the equilibrium ($n_1^e < n_1^{opt}$).

4.2 The strategic tax rates are not efficient. In the case of $b < (\beta - \gamma)$, both governments provide the public good over the optimal level ($G_i^e < G_i^{opt}$, $i = 1,2$). However, in the case of $b > (\beta - \gamma)$, it becomes that $G_1^e < G_1^{opt}$, $G_2^e > G_2^{opt}$.

4.3 The housing cost is lower than optimal level in city 1, while higher in city 2 regardless of b.

6. Concluding remarks

In the traditional tax competition literature, the economies of respective regions are assumed to be symmetric or partially asymmetric and the firms of respective regions perform all production in their regions. It is also not unusual that the work place is not necessarily located to the workers' residence in a metropolitan area.

We considered a metropolitan area which consists of a central city and its surrounding city, and applied the tax competition to the metropolitan area. Our model allowed for all the residents in the metropolitan area to work at the central city even though some of them reside in the surrounding city. Moreover, we introduced the proportional income tax instead of capital tax instead of capital tax.

The main results of our analysis are as follows. In the closed economy, each government has a dominant strategy in determining its tax rate, and tax rate in city 1 is higher than that of city 2. In an open economy, the housing lot's elasticity of utility (β) should be higher than that of the public good (γ) for the stable migration equilibrium. Otherwise, city 1 or city 2 will become completely depopulated.

Second, while the tax rate of city 2 is lower than that in closed economy regardless of marginal productivity of public good (b), the tax rate of city 1 is lower (higher) than that in closed economy when $b > (<)(\beta - \gamma)$. Finally, comparing to the optimal allocation, less concentration to the central city occurs in equilibrium, and the provision of public good is not efficient. When the productivity of the public good is relatively low, both governments provide the public good over the optimal level. However, it is relatively high, city 1 provides the public good at a less than the optimal level, while city 2 over provides. It is not rare that tax competition leads to the overprovision of public good, e.g.

Keen and Marchnad (1997). We should pay attention to the case of $b > (\beta - \gamma)$. In this case, the underprovision of public good in city 1 occurs even though the tax rate is higher than the optimal level. As we consider the proportional income tax, the provision of public good depends on the wage and population. In equilibrium, the population in city 1 is significantly less than the optimal level. Therefore, the underprovision of public good occurs in city 1 though the tax rate is higher than the optimal level.

The main goal of this paper is analyzing governments' tax policy in the metropolitan area in which all the firms are located in the central city. Considering suburbanization or edge city, a possible extension of this model could be the free mobility of firms just like the residents. It would be interesting to analyze the interactions between cities due to the change of firms' location. In this case, it is necessary to consider not only the income tax to the residents but also the tax to the firms. Further research should investigate these problems.

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